КАФЕДРА ВЫСШЕЙ МАТЕМАТИКИ

МИНИСТЕРСТВО ОБРАЗОВАНИЯ И НАУКИ РОССИЙСКОЙ ФЕДЕРАЦИИ
ФГБОУ ВПО «МОСКОВСКИЙ ГОСУДАРСТВЕННЫЙ СТРОИТЕЛЬНЫЙ УНИВЕРСИТЕТ»

А.Г. Мясников, А.Н. Серова

ЭЛЕМЕНТЫ ТЕОРИИ ПОЛЯ

Варианты заданий для самостоятельной работы студентов

По направлениям «Прикладная математика»,
«Прикладная механика»

Задача 1. Используя формулу Остроградского-Гаусса, вычислить поток векторного поля **a**(x,y,z) через поверхность G цилиндрического тела, ограниченного сверху графиком функции z = f(x,y), а снизу – областью D на координатной плоскости xOy.

1) $f(x,y) = (x-1)^2+y^2$, D: $x+y = 1$, $x = 0$, $y = 0$, $\mathbf{a}(x,y,z) = x\mathbf{i}+3y\mathbf{j}-z\mathbf{k}$.
2) $f(x,y) = (4-x^2-y^2)^{1/2}$, D: $x^2+y^2 = 4$, $\mathbf{a}(x,y,z) = 3x\mathbf{i}-2y\mathbf{j}+z\mathbf{k}$.
3) $f(x,y) = \sin^2 x$, D: $x = 0$, $x = \pi/2$, $y = 0$, $y = \cos x$,
$\mathbf{a}(x,y,z) = \cos x\mathbf{i}+(x+y)\mathbf{j}+z\sin x\mathbf{k}$.
4) $f(x,y) = (x^2+y^2+1)^{1/2}$, D: $x^2+y^2 = 3$, $x = 0$ $(x\le 0)$,
$\mathbf{a}(x,y,z) = x\mathbf{i}-y\mathbf{j}+(z+(x^2+y^2+1)^{1/2})\mathbf{k}$.
5) $f(x,y) = \cos x$, D: $y = 0$, $y = \cos x$ $(-\pi/2\le x\le \pi/2)$,
$\mathbf{a}(x,y,z) = 2x\mathbf{i}+xy\mathbf{j}-xz\mathbf{k}$.
6) $f(x,y) = x^2+y^2$, D: $x^2+y^2 = 4$, $x = 0$, $y = 0$ $(x\ge 0, y\ge 0)$,
$\mathbf{a}(x,y,z) = (x-y)\mathbf{i}+(x+y)\mathbf{j}+(z-y)\mathbf{k}$.
7) $f(x,y) = 2x^2+(y-3)^2$, D: $x = -1$, $x = 0$, $y = 0$, $y = x+3$,
$\mathbf{a}(x,y,z) = 2x\mathbf{i} + xz\mathbf{j}-3z\mathbf{k}$.
8) $f(x,y) = (1+x^2+y^2)$, D: $x^2+y^2 = 1$, $x = 0$, $y = 0$ $(x\le 0, y\ge 0)$,
$\mathbf{a}(x,y,z) = (2x+y)\mathbf{i}+(2y-x)\mathbf{j}+(x^2+y^2)\mathbf{k}$.
9) $f(x,y) = (1-x)x$, D: $x = 1$, $y = 0$, $y = x$, $\mathbf{a}(x,y,z) = xz\mathbf{i}-yz\mathbf{j}+z\mathbf{k}$.
10) $f(x,y) = (x^2+y^2+1)^{½}$, D: $x^2+y^2 = 4$, $x = 0$ $(x\ge 0)$,
$\mathbf{a}(x,y,z) = (x+y)\mathbf{i}-(x+y)\mathbf{j}+2z\mathbf{k}$.
11) $f(x,y) = e^{-x^2}$, D: $x = 0$, $x = 1$, $y = 0$, $y = x$,
$\mathbf{a}(x,y,z) = 0.5x^2\mathbf{i}-xy\mathbf{j}+z\mathbf{k}$.
12) $f(x,y) = 4-x^2-y^2$, D: $x^2+y^2 = 4$, $y = 0$ $(y\ge 0)$, $\mathbf{a}(x,y,z) = 2x\mathbf{i}+y\mathbf{j}-2z\mathbf{k}$.
13) $f(x,y) = x+y+2$, D: $x = 0$, $y = 0$, $y = 1-x^2$ $(x\ge 0)$,
$\mathbf{a}(x,y,z) = (x+y+z)\mathbf{i}+(x+z)\mathbf{j}-(x+y)\mathbf{k}$.
14) $f(x,y) = 1+x^2+y^2$, D: $x^2+y^2 = 1$, $y = 0$ $(y\le 0)$,),
$\mathbf{a}(x,y,z) = -x\mathbf{i}+x^3 j\mathbf{k}+2z\mathbf{k}$.
15) $f(x,y) = 2 \sin x/2 \cos x/2$, D: $y = \sin x$, $y = 0$ $(0\le x\le \pi)$,
$\mathbf{a}(x,y,z) = x\mathbf{i}+xyz\mathbf{j}-0.5xz^2\mathbf{k}$.
16) $f(x,y) =2-(x^2+y^2)^{½}$, D: $x^2+y^2 = 1$, $x^2+y^2 = 4$,
$\mathbf{a}(x,y,z) = 0.5x^2y\mathbf{i}+y^2\mathbf{j}-xyz\mathbf{k}$.
17) $f(x,y) = e^x$, D: $x = 0$, $x = 1$, $y = 0$, $y = 2-x$,
$\mathbf{a}(x,y,z) = (x+yz)\mathbf{i}+e^y\mathbf{j}+(xy-z)\mathbf{k}$.
18) $f (x,y) = (x^2+y^2)^{½}$, D: $x^2+y^2 = 4$, $x^2+y^2 = 9$, $y = 0$ $(y\le 0)$,
$\mathbf{a}(x,y,z) = x\cos y\mathbf{i}-\sin y\mathbf{j}+z\mathbf{k}$.
19) $f(x,y) = y(y-1)$, D: $x = 0$, $y = 0$, $y = x+1$,
$\mathbf{a}(x,y,z) =(-x+y+z)\mathbf{i}+ \ln y\mathbf{j}+(x+y+z)\mathbf{k}$.
20) $f(x,y) = 4-x^2-y^2$, D: $x^2+y^2 = 1$, $x^2+y^2 = 4$, $y = 0$ $(y\ge 0)$,
$\mathbf{a}(x,y,z) = x^2\mathbf{i}+ye^x\mathbf{j}-ze^x\mathbf{k}$.
21) $f(x,y) = e^2-x$, D: $x = e$, $x = e^2$, $y=0$, $y = 1/\ln x$,
$\mathbf{a}(x,y,z) = xy^2\mathbf{i}+y\ln x\mathbf{j}-y^2z\mathbf{k}$.
22) $f(x,y) = (4x^2+4y^2)^{½}$, D: $(x-1)^2+y^2 = 1$, $y = 0$ $(y\ge 0)$,

$\mathbf{a}(x,y,z) = xy\mathbf{i}+xz\mathbf{j}+yz\mathbf{k}$.

23) $f(x,y) = x+2y$, D: $x = 0$, $x = 1$, $y = 0$, $y = \sqrt[3]{x}$,
$\mathbf{a}(x,y,z) = x(y+z)\mathbf{i}+y^2\mathbf{j}-(y+0.5z)z\mathbf{k}$.

24) $f(x,y) = (16-x^2-y^2)$, D: $x^2+y^2 = 1$, $x^2+y^2 = 4$, $x = 0$, $y = 0$ $(x\geq 0, y\geq 0)$, $\mathbf{a}(x,y,z) = x\mathbf{i}-y\mathbf{j}+xz\mathbf{k}$.

25) $f(x,y) = 4-x^2$, D: $y = x^2$, $y = 1$, $\mathbf{a}(x,y,z) = x^3\mathbf{i}-e^y\mathbf{j}+ze^y\mathbf{k}$.

26) $f(x,y) = (1-x^2-y^2)^{1/2}$, D: $x^2+y^2 = 1$, $y = -x$, $y = x$ $(x\geq 0)$,
$\mathbf{a}(x,y,z) = xyz\mathbf{i}+0.5y^2z\mathbf{j}+x\mathbf{k}$.

27) $f(x,y) = 3-x-y$, D: $x = 1$, $y = 0$, $y = x^{1/2}$,
$\mathbf{a}(x,y,z) = x\ln(y+1)\mathbf{i}+2y\mathbf{j}-z\ln(y+1)\mathbf{k}$.

28) $f(x,y) = (2x^2+2y^2)$, D: $x^2+y^2 = 1$, $x^2+y^2 = 4$, $x = 0$, $y = 0$ $(x\leq 0, y\geq 0)$,
$\mathbf{a}(x,y,z) = (x^2+y^2)^{-1}\mathbf{i}-(x^2+y^2)^{-1}\mathbf{j}$.

29) $f(x,y) = \cos^2 x$, D: $y = 0$, $y = \mathrm{tg}x$ $(0\leq x\leq \pi/4)$,
$\mathbf{a}(x,y,z) = \sin x\mathbf{i}+y\mathbf{j}-z\mathbf{k}$.

30) $f(x,y) = (1-y)^{1/2}$, D: $x = 0$, $y = x$, $y = 1$,
$\mathbf{a}(x,y,z) = (x+y^2)\mathbf{i}+(x^2-y)\mathbf{j}+z^2\mathbf{k}$.

Задача 2. Двумя способами (с помощью криволинейного интеграла 2-го рода и с помощью формулы Стокса) найти циркуляцию векторного поля $\mathbf{a}(x,y,z)$ вдоль линии пересечения Г поверхности G с координатными плоскостями при $x\geq 0$, $y\geq 0$, $z\geq 0$.

1) G: $z = 1-y-x^2$, $\mathbf{a}(x,y,z) = (x-y)\mathbf{i}+(2x+y)\mathbf{j}$.
2) G: $z = 1/4(4-x-y)^{1/2}$, $\mathbf{a}(x,y,z) = y^2\mathbf{i}+z\mathbf{j}+y\mathbf{k}$.
3) G: $z = 1-(x+y)^{1/2}$, $\mathbf{a}(x,y,z) = 2y^2\mathbf{i}+3x^2\mathbf{j}+\mathbf{k}$.
4) G: $z = 4-4x^2-y$, $\mathbf{a}(x,y,z) = (2x+3y)\mathbf{i}+(z-3y)\mathbf{k}$.
5) G: $z = (1-x)^{1/2}-y$, $\mathbf{a}(x,y,z) = (x+y)\mathbf{i}-\mathbf{j}$.
6) G: $z= 1-x-y^2$, $\mathbf{a}(x,y,z) = (x-y+z)\mathbf{k}$.
7) G: $z = (4-y)^{1/2}-x$, $\mathbf{a}(x,y,z) = 2x\mathbf{i}-z\mathbf{j}+y\mathbf{k}$.
8) G: $z = (1-x-y^2)^{1/2}$, $\mathbf{a}(x,y,z) = y\mathbf{i}-x\mathbf{j}+z\mathbf{k}$.
9) G: $z = 4-x^2-y$, $\mathbf{a}(x,y,z) = y\mathbf{i}-x^2\mathbf{j}+z^2\mathbf{k}$.
10) G: $z = (4-x-y)^{1/2}$, $\mathbf{a}(x,y,z) = x^2\mathbf{j}+z\mathbf{k}$.
11) G: $z = 4-(x+y)^2$, $\mathbf{a}(x,y,z) = -z\mathbf{i}+y\mathbf{j}+x\mathbf{k}$.
12) G: $z = 4-x-y^2$, $\mathbf{a}(x,y,z) = 2x\mathbf{i}+z\mathbf{j}+x\mathbf{k}$.
13) G: $z = (1-y-x^2)^{1/2}$, $\mathbf{a}(x,y,z) = y^2\mathbf{i}-x^2\mathbf{j}+z^2\mathbf{k}$.
14) G: $z = 4-x-4y^2$, $\mathbf{a}(x,y,z) = x^2\mathbf{j}-(y+1)\mathbf{k}$.
15) G: $z = (1-x-y)^{1/2}$, $\mathbf{a}(x,y,z) = (x-y+z)\mathbf{i}+x\mathbf{j}$.
16) G: $2x^{1/2}+y+z = 2$, $\mathbf{a}(x,y,z) = (2x+y)\mathbf{i}-z\mathbf{j}+(x+z)\mathbf{k}$.
17) G: $(2x+z)^2+y = 9$, $\mathbf{a}(x,y,z) = z\mathbf{i}+(x-y)\mathbf{j}+(x+2z)\mathbf{k}$.
18) G: $x+y^{1/2}+z^{1/2} = 1$, $\mathbf{a}(x,y,z) = z\mathbf{i}+2x\mathbf{j}+x\mathbf{k}$.
19) G: $x^2+y+z^2 = 4$, $\mathbf{a}(x,y,z) = (x+y+z)\mathbf{k}$.
20) G: $x^{1/2}+y+z^{1/2} = 1$, $\mathbf{a}(x,y,z) = x^{1/2}\mathbf{j}+z^{1/2}\mathbf{k}$.
21) G: $x^2+y^2+z = 4$, $\mathbf{a}(x,y,z) = xy\mathbf{i}+z^2\mathbf{j}$.
22) G: $12x^2+3y+4z = 24$, $\mathbf{a}(x,y,z) = (z-y)\mathbf{i}=(2x+y)\mathbf{j}$.
23) G: $(2x+y)^2+z = 9$, $\mathbf{a}(x,y,z) = (x+2z)\mathbf{i}+(y-x)\mathbf{k}$.
24) G: $(x+y)^2+z = 1$, $\mathbf{a}(x,y,z) = (2x+z)\mathbf{i}+y\mathbf{j}-z\mathbf{k}$.

25) G: $(x^2+y^2)^{1/2}+z = 1$, $\mathbf{a}(x,y,z) = y^2\mathbf{j}+x^2\mathbf{k}$.
26) G: $x^2+2y+2z = 4$, $\mathbf{a}(x,y,z) = xy\mathbf{i}+z\mathbf{j}+4y\mathbf{k}$.
27) G: $(x+2y)^2+z = 9$, $\mathbf{a}(x,y,z) = (x+y)\mathbf{i}+x\mathbf{j}+2y\mathbf{k}$.
28) G: $x+2y+z^{1/2} = 2$, $\mathbf{a}(x,y,z) = (x+y)\mathbf{i}+(x-y)\mathbf{j}+(x+y)\mathbf{k}$.
29) G: $x+2y^{1/2}+z = 2$, $\mathbf{a}(x,y,z) = (x+z)\mathbf{i}+(x+y)\mathbf{j}$.
30) G: $x+y+z^{1/2} = 1$, $\mathbf{a}(x,y,z) = xz\mathbf{i}+x\mathbf{j}-2z\mathbf{k}$.

Задача 3. При каких значениях параметров α, β, γ векторное поле $\mathbf{a}(x,y,z)$ является а) соленоидальным; б) потенциальным; в) гармоническим ?

1) $\mathbf{a}(x,y,z) = (\alpha x+\beta y+z)\mathbf{i}+(\gamma x+\alpha y)\mathbf{j}+(\beta x+z)\mathbf{k}$.
2) $\mathbf{a}(x,y,z) = (\beta x+y)\mathbf{i}+(\beta x-\alpha y+\gamma z)\mathbf{j}+(y+\beta z)\mathbf{k}$.
3) $\mathbf{a}(x,y,z) = (\alpha x+y)\mathbf{i}+(-2\gamma x-2\beta y+z)\mathbf{j}+(-\beta y+\gamma z)\mathbf{k}$.
4) $\mathbf{a}(x,y,z) = \beta x\mathbf{i}+(\alpha x+3y-z)\mathbf{j}+\gamma y\mathbf{k}$.
5) $\mathbf{a}(x,y,z) = (2\gamma x+3z)\mathbf{i}+(\gamma y+\beta z)\mathbf{j}+(-\alpha x+y+\alpha z)\mathbf{k}$.
6) $\mathbf{a}(x,y,z) = (3\alpha x-\gamma y+2z)\mathbf{i}+(2x+\beta y)\mathbf{j}+(-\alpha x-2\gamma z)\mathbf{k}$.
7) $\mathbf{a}(x,y,z) = (\alpha x+\gamma z)\mathbf{i}+(2\alpha y+3z)\mathbf{j}+(x+\beta y+z)\mathbf{k}$.
8) $\mathbf{a}(x,y,z) = (\alpha x-y+\beta z)\mathbf{i}+(\gamma x-2\alpha y)\mathbf{j}+(3x-\gamma z)\mathbf{k}$.
9) $\mathbf{a}(x,y,z) = (3\beta x+2\beta z)\mathbf{i}+(-2\alpha y+z)\mathbf{j}+(-x+2\alpha y+\gamma z)\mathbf{k}$.
10) $\mathbf{a}(x,y,z) = (\gamma x-2\beta z-2y)\mathbf{i}+(\alpha x-y)\mathbf{j}+(x-z)\mathbf{k}$.
11) $\mathbf{a}(x,y,z) = (x+\beta y)\mathbf{i}+(3x-\gamma y+2\alpha z)\mathbf{j}+(y+3\beta z)\mathbf{k}$.
12) $\mathbf{a}(x,y,z) = (2\alpha x+3\gamma y)\mathbf{i}+(x+\beta y+2z)\mathbf{j}+(\alpha y-\gamma z)\mathbf{k}$.
13) $\mathbf{a}(x,y,z) = (3\gamma x+\alpha y-z)\mathbf{i}+(-2x+\gamma y+\beta z)\mathbf{j}+(-x+y+2z)\mathbf{k}$.
14) $\mathbf{a}(x,y,z) = (\beta x-2z)\mathbf{i}+(2\alpha y-z)\mathbf{j}+(\alpha x+\gamma y+z)\mathbf{k}$.
15) $\mathbf{a}(x,y,z) = (-2\beta x+\gamma z)\mathbf{i}+(\alpha y+2z)\mathbf{j}+(x+\beta y+3\gamma z)\mathbf{k}$.
16) $\mathbf{a}(x,y,z) = (\alpha^2 x+2z)\mathbf{i}+(-3\alpha y-z)\mathbf{j}+(2\gamma x+\beta y+2z)\mathbf{k}$.
17) $\mathbf{a}(x,y,z) = (\beta x+2y)\mathbf{i}+(\alpha x-2\beta y+z)\mathbf{j}+(\gamma^2 y+z)\mathbf{k}$.
18) $\mathbf{a}(x,y,z) = (\alpha x-2\beta z)\mathbf{i}+(\beta^2 y+\alpha z)\mathbf{j}+(\beta^2 x+\alpha y-z)\mathbf{k}$.
19) $\mathbf{a}(x,y,z) = -6x\mathbf{i}+(\beta^2 y+\gamma z+z)\mathbf{j}+(2\beta^2 y-\beta z)\mathbf{k}$.
20) $\mathbf{a}(x,y,z) = (-\beta x-y)\mathbf{i}+(\gamma x+\alpha y+16z)\mathbf{j}+(\beta^2 y-\gamma z)\mathbf{k}$.
21) $\mathbf{a}(x,y,z) = (4x-\gamma y+\alpha z)\mathbf{i}+(x+\beta^2 y)\mathbf{j}+(x-5\beta z)\mathbf{k}$.
22) $\mathbf{a}(x,y,z) = (\gamma x-\beta y)\mathbf{i}+(x+2\gamma y+2\alpha z+3z)\mathbf{j}+(\alpha^2 y-z)\mathbf{k}$.
23) $\mathbf{a}(x,y,z) = (-\gamma^2 x+y)\mathbf{i}+(x+2y-5\gamma z+6z)\mathbf{j}+(\gamma^2 y+\beta z)\mathbf{k}$.
24) $\mathbf{a}(x,y,z) = (\gamma^2 z+z)\mathbf{i}-3\gamma^2 y\mathbf{j}+(\alpha x+75z)\mathbf{k}$.
25) $\mathbf{a}(x,y,z) = (\gamma x+y+9z)\mathbf{i}+(\beta x-2\beta y)\mathbf{j}+(\alpha^2 x-2\alpha z)\mathbf{k}$.
26) $\mathbf{a}(x,y,z) = (\gamma^2 x+y)\mathbf{i}+(2\alpha x+3z)\mathbf{j}+(\beta y-2\gamma z)\mathbf{k}$.
27) $\mathbf{a}(x,y,z) = (x+2y+4\beta z-3z)\mathbf{i}+(\gamma x-2\alpha y)\mathbf{j}+(\beta^2 x-2\alpha z)\mathbf{k}$.
28) $\mathbf{a}(x,y,z) = (\gamma x+\alpha^2 y)\mathbf{i}+(4x-\alpha^2 y)\mathbf{j}+3z\mathbf{k}$.
29) $\mathbf{a}(x,y,z) = (-2\alpha x-y+\beta y)\mathbf{i}+(\alpha^2 x+\alpha x-8y)\mathbf{j}+\alpha^2 z\mathbf{k}$
30) $\mathbf{a}(x,y,z) = (\alpha x+3\gamma z-\gamma^2 z)\mathbf{i}+(-2\gamma y-z)\mathbf{j}+(-4x+\alpha y-\beta z)\mathbf{k}$.